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COMMENT

Comment on 'Anomalous dynamics of unbiased polymer translocation through a narrow pore' and other recent papers by D Panja, G Barkema and R Ball

J L A Dubbeldam[1,2]**, A Milchev**[1,3]**, V G Rostiashvili**[1] **and T A Vilgis**[1]

[1] Max Planck Institute for Polymer Research, 10 Ackermannweg 55128 Mainz, Germany
[2] Delft University of Technology 2628CD Delft, The Netherlands
[3] Institute for Physical Chemistry Bulgarian Academy of Science, 1113 Sofia, Bulgaria

Abstract
In a recent publication of Panja *et al* (2007 *J. Phys.: Condens. Matter* **19** 432202) they suggested a new interpretation of the translocation problem of polymer chain threading through a narrow pore. Here we point out some contradictions and inconsistencies in this treatment which question the plausibility of the obtained results.

In a recent publication Panja *et al* [1] presented a new theoretical interpretation of the problem of unbiased translocation through a narrow pore in a membrane. The authors were mainly dealing with the scaling law for the average translocation time τ_d in terms of the chain length N. The time τ_d is defined as an average first-passage time of the translocation process. They also performed MC-simulations to test their theoretical predictions. Their considerations were based on linear response theory with memory effects and the fluctuation-dissipation theorem (FDT) which, as the authors assumed, are applicable to monomers threading through the pore. The main results were the following. The mean-square displacement of the translocation coordinate s is subdiffusive at times $t \leqslant \tau_R$ (where τ_R is the characteristic Rouse time) and then it crosses over to a normal diffusive regime at $t \geqslant \tau_R$.

Namely, one has

$$\langle s^2 \rangle \propto \begin{cases} t^{\frac{1+\nu}{1+2\nu}}, & t < \tau_R \\ D_N t, & t > \tau_R \end{cases} \tag{1}$$

where ν is the Flory exponent and D_N is a diffusion coefficient which is discussed below. As a result of this anomalous dynamics, the average translocation timescales as

$$\tau_d \propto N^{2+\nu}. \tag{2}$$

The authors emphasize that τ_d is *larger* than τ_R so that the scaling given by equation (2) differs from $\tau_R \propto N^{1+2\nu}$ (we recall here the seminal paper of Chuang *et al* [2] where it was argued that despite $\tau_d > \tau_R$, the translocation time τ_d should scale as $\tau_d \propto N^{1+2\nu}$, i.e. the presence of a constraint (membrane) does not affect the τ_d versus N dependence).

In this comment we would like to point out some contradictions in the theoretical treatment given in [1] which question the validity of the employed approach and its main conclusions.

As a first general remark one should mention that in the treatment, presented in [1], the membrane is present only as an imaginary separating plane between the left (*cis*), and right (*trans*) semi-spaces. Nowhere in the text is the proper statistics of the polymer chain, having one of its ends tethered at the *impenetrable* membrane, considered. Therefore, it remains unclear why the resulting scaling for τ_d differs from that for $\tau_R \propto N^{1+2\nu}$. The memory effects, which the authors refer to, exist also in the case of an imaginary membrane.

The authors handle the problem in terms of the linear response theory which links the chain tension $\phi(t)$ and the translocation velocity $v(t) = \dot{s}(t)$ by means of

$$\phi(t) = \int_0^t \mu(t - t')v(t') \, dt' \tag{3}$$

where $\mu(t)$ is a memory kernel which, according to FDT, can be expressed through the autocorrelation function $\mu(t - t') = \langle \phi(t)\phi(t') \rangle_{v=0}$. The inversion of equation (3) yields

$$v(t) = \int_0^t a(t - t')\phi(t') \, dt', \qquad (4)$$

where again, according to FDT, $a(t-t') = \langle v(t)v(t') \rangle_{\phi=0}$, and the relation between the respective Laplace transforms reads

$$\tilde{\mu}(k) = \tilde{a}^{-1}(k) \qquad (5)$$

with k being the Laplace conjugate variable.

The calculation of $\mu(t)$ is the central point of [1]. To this end they suggest the following *Gedanken* experiment. Let us assume that a chain portion n_t (tethered with one end at the membrane) is equilibrated at the left-hand side (lhs) of the membrane. Then suddenly p monomers are injected from the lhs to the right-hand side (rhs) of the membrane which corresponds to an impulse current $v(t) = p\delta(t)$. According to equation (3), the memory kernel will reflect the tension relaxation $\phi(t)$. Assuming that $p \ll n_t$ and that the characteristic relaxation time goes as $t \propto n_t^{1+2\nu}$, one can then prove that the memory kernel $\mu(t) \propto t^{-\frac{1+\nu}{1+2\nu}}$. Using the Laplace transform of the velocity correlation function equation (5), and the expression for the mean-square displacement, $\langle s^2(t) \rangle = 2 \int_0^t (t - t') \langle v(t)v(t') \rangle \, dt'$, one obtains the result given in equation (1). The authors claim that at $t > \tau_R$ the process follows the law of normal diffusion so that the matching of the subdiffusive and diffusive regimes at $t = \tau_R$ leads to $D_N \tau_R = \tau_R^{\frac{1+\nu}{1+2\nu}}$, or $D_N \propto \tau_R^{-\frac{\nu}{1+2\nu}}$. Taking into account that $\tau_R \propto N^{1+2\nu}$, one obtains $D_N \propto N^{-\nu}$. After a period equal to the translocation time τ_d one has $\langle s^2(\tau_d) \rangle = N^2$, i.e. (see equation (1)) $D_N \tau_d = N^2$, or $\tau_d \propto N^{2+\nu}$, as suggested by equation (2).

It is strange that this result implies $D_N \propto N^{-\nu}$ (as in the case of Zimm dynamics with hydrodynamic interactions which are absent in this model!). It turns that the chain diffusion coefficient is much larger than that for a Rouse unconstrained chain, $D_R \propto N^{-1}$, although on the other hand, one claims that $\tau_d \geqslant \tau_R$. Therefore, one might wonder whether a crossover to normal diffusion at time $t \approx \tau_R$ really exists. Note, that such a crossover (manifested by a *knick*-point in $\langle s^2(t) \rangle$) has actually never been seen in the numerous previous computer experiments on translocation dynamics. It is not detected even in figure 3 of the paper of Panja *et al* [1]. In contrast, at sufficiently late times $\langle s^2(t) \rangle$ is observed to go to a plateau, as it should, because the largest possible value of the translocation coordinate s is always strictly limited by the chain length N, i.e. $0 \leqslant s \leqslant N$, despite the broad distribution of translocation times.

The linear response theory has also been utilized in the case of field-driven translocation in another paper by the authors [3] where the chain is subjected to a driving force E, acting inside the pore. Under this condition the chain tension imbalance between the left and the right sides of the pore varies almost linearly in E (see figure 4 in [3]), i.e. $\phi_{t=0} - \phi(t) \propto E$. From equation (7) in [3] one obtains for the velocity $v(t) \propto$

$Et^{-\frac{\nu}{1+2\nu}}$ and for the translocation coordinate $s(t) \propto Et^{\frac{1+\nu}{1+2\nu}}$. The condition $s(\tau_d) = N$ then yields

$$\tau_d \propto \left(\frac{N}{E} \right)^{\frac{1+\nu}{1+\nu}}. \qquad (6)$$

A comparison with equation (8) in [3] (which reads $\tau_d \propto N^{\frac{1+\nu}{1+2\nu}}/E$ and where the E-dependence was only numerically obtained) shows that the linear response method fails to predict the correct E-dependence of the translocation time. The authors faced similar problems in their study of the translocation dynamics by pulling a chain through a pore with a force F applied to one end of the polymer [4]. In this case it is well known [5] that at $N^{-\nu} \leqslant aF/k_B T \leqslant 1$ (where a is the Kuhn segment length and T is the temperature) the chain reacts *nonlinearly* to applied force (the so-called Pincus scaling law). Thus one may wonder to what degree is the linear response theory an adequate tool for handling the problem. Moreover, the scaling expression $\tau_d \propto N^{\frac{1+\nu}{1+2\nu}}/E$ is inconsistent with the natural scaling form $\tau_d = N^{2+\nu} f(EN)$ where EN is the work performed by the field E during the time τ_d.

Finally, we should comment on the criticism [1] of our recent publication [6], in which we studied the dynamics of unbiased translocation. We recall that our consideration is based on the notion of a *fold*, an equilibrated portion of the chain which threads through the pore whereas the total chain is far from equilibrium. The fold experiences an entropic barrier while it is threading randomly between the *cis*- and *trans*-sides of the membrane. We have shown that overcoming of the entropic barrier leads to anomalous diffusion in the translocation length space, namely $\langle s^2(t) \rangle \propto t^\alpha$ where $\alpha = 2/(2\nu + 2 - \gamma_1)$ and γ_1 is the surface exponent ($\gamma_1 = 0.68$ in three-dimensional space). We have assumed that the proper fractional diffusion equation (FDE) with the corresponding anomalous diffusion exponent $\alpha = 2/(2\nu + 2 - \gamma_1)$ could be used for the complete description of the translocation process in terms of probability distribution function $P(s, t)$. The FDE-formalism is very convenient because it enables one to analyze the first-passage time distribution function $Q(t)$ (which is related to $P(s, t)$) as well as to impose some particular (e.g. reflecting at $s = 0$ and adsorbing at $s = N$) boundary conditions. As demonstrated in [6], the results from our dynamic Monte Carlo study perfectly agree with the theoretical predictions.

It appears to us that the criticism in [1], concerning the entropic barrier contribution, is totally based on a misunderstanding. Indeed, it is claimed [1] that repeated occurrence of the state $s = 0$ with the eventual reaching of the state $s = N$ gives rise to an entropic barrier peaked at $s = N/2$. It should be emphasized that the return to the state $s = 0$ many times before eventually the state $s = N$ is reached has nothing to do with a barrier! As explained above, the entropic barrier is assigned to a fold which is much shorter than the chain length. This notion implies local equilibration on the scale of the fold and makes it possible to use (locally) an expression for the free energy. Moreover, the particular choice of boundary conditions at $s = 0$ and N (reflecting or adsorbing) plays a minor role in the calculation of the

first-passage time distribution function $Q(t)$ as well as of the first $\langle s(t) \rangle$ and the second $\langle s^2(t) \rangle$ moments of the probability distribution function $P(s, t)$. Therefore, the criticism raised in [1] is in our opinion not well founded.

References

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